MECHANICS OF SOLIDS

Brainstorming seminars at the
Budapest University of Technology and Economics

Editor: György Károlyi
Cover design: Mónika Fehér

Mechanics of Solids,
Brainstorming Seminars at the Budapest University of Technology and Economics
2000
ISBN 963 420 669 7

Printed by the Publishing Company of the
Budapest University of Technology and Economics

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PREFACE

Two PhD students, Krisztina Polgár and Árpád Meggyes, initiated the Solid Mechanics Seminars in 1998. This lecture series aims at giving an opportunity for PhD students and young scientists to present the results of their research to a friendly but critical audience, and to foster new ideas through the lectures given by more experienced researchers and professors. The seminars are held at the Department of Applied Mechanics and at the Department of Structural Mechanics of the Budapest University of Technology and Economics in an alternating manner to build a closer connection between our departments. The main intention was, however, and still is to draw together people working in a wide variety of interesting fields related to solid mechanics. This allowed us to learn from each other and also to initiate open discussions on the topics covered by the lectures. Our hope is that it was fruitful for all those who spent their valuable time with us either by giving presentation or by attending them. We also hope that we could connect people working on different research areas so that they could further exchange ideas and promote their scientific results. If only part of these goals have been achieved, we are satisfied, our efforts were not futile.

We are grateful for those who volunteered to give fascinating talks of high scientific level. We also have to thank everybody who showed interest, came, and spent his time with us. We hope it was worthwhile, and we also hope that the remarks, sometimes provocative discussions, but the always warm atmosphere was also beneficial for the lecturers. We are grateful to the Research Relations Department of the Technical University of Budapest for the financial support. The publication of this proceedings of ten abstracts would not be possible without their funding.

We hope to further increase the interest in our seminars in the future. Fortunately, we are not lacking volunteering lecturers from various topics of mechanics, which can attract a broad audience.

The three years of successful lectures could not be interrupted by the sudden death of Árpád Meggyes, one of the organizers and initiators of these seminars. This indicates how well he noticed the lack of such opportunities at our university. This collection of lecture summaries is dedicated to the memory of Árpád Meggyes.

Budapest, February 2001.

Katalin Bagi Department of Structural Mechanics	György Károlyi Department of Structural Mechanics

Mechanics of Solids, 21st March 2000
Brainstorming Seminars at the
Budapest University of Technology and Economics

KINEMATICAL DEGREE OF FREEDOM OF CLASSICAL AND GENERALIZED CONTINUUM

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Keywords: continuum mechanics, classical, generalized and lattice continuum, micropolar theory of elasticity, theory of directories.

1991 Mathematics Subject Classification: 74A30.

Summary: The degrees of freedom of a continuum's points are the three components of translation vector. Can the degrees of freedom be increased by taking into account three independent components of the rotation vector in such a way, that the continuum stays continuum? The degrees of freedom of rigid bodies at lattice points (of lattice continuum) are the three-three components of translation and rotation vector. Does the limit of continuum exist and keep all six degrees of freedom? In the case of structured—micropolar—material, can we form continuum by evaluating the limit of kinematical degrees of freedom definied in the cell—translation, rotation, various forms of deformation—such, that those become independent degrees of freedom of continuum? In the lecture we try to answer these question.

Definition: From the kinematical point of view, *classical continuum* is a (three-dimensional) domain of a (three-dimensional) Riemann-space. By the definition, the kinematical degree of freedom of a continuum is the translation (vector) field. The translation field creats the rotation (vector) field unambiguosly. That is, the continuum does not have independent rotation field.

Inversely, independent rotation field in the continuum can not be created. Namely, the elements of a continuum are points without size and dimension, that is any basis can not be assigned to the element itself. The basis—natural tangent trieder—that can be assigned to the points of continuum does not rotate—can not rotate—independently from translation. The cause of it is that the analysed domain is a domain of a Riemann-space. At the same time an independent trieder can not be assigned to the point, because the point is without size and dimension, and to define the trieder we need at least three points (which gives two directions, and the third direction is created by cross-product).

The lack of size and dimension of points, and the differentiable manifold, as a structure, are in close connection. The basic set of the structure is a (point) manifold, the topology given on it guarantees the continuity, the differentiable structure guarantees the suitable smoothness. The different density functions allow for defining the different physical quantities—mass, surface and volume force.

To sum it up: the kinematical degree of freedom in the classical continuum is the translation field, the translation field determines the rotation field unambiguosly, and independent rotation field, like kinematical degree of freedom, can not be introduced.

Continuous modelling is based on the mathematical idea of manifolds, differentiable manifolds, Riemann-space. By the explanation of this idea the following theorem, and its reversion, can be proved.

Theorem 1: In the Riemann-space only the classical continuum exists.

The reversion of the Theorem 1: The classical continuum exists only in the Riemann-space.

Let a (finite or infinite) periodical lattice be embedded in the (three-dimensional) Euclidean space with bodies of the same mechanical properties in every point of the periodic lattice, and assume given kinematical and dynamical connections between the elements, which are identical in case of identical relative positions of two elements. The mechanical system defined above is called (absolute) periodical lattice continuum. This continuum is a discrete system. In the case of a discrete problem, e.i. lattice continuum—except in case of continuos series expansion—the following contradictions can be found:

- the differentiation can not be done in the coordinate of embedding space,
- six- or more dimensional basis is used in the three dimensional space,
- the stress-couple tenzor can not be interpreted.

For the existence of lattice continuum the elements of the analysed set must not be point like, they must have finite size. In this case the set can contain only countable many elements, so only discrete topology can be introduced on it. The individual points are not accumulation points, this is why the differentations can not be performed. The continuous tools are inapplicable for describing the system.

By decreasing the size of the set's elements we can add only finite-size elements to the system. Evaluating the limit by these steps we add countable many countable set, which is also countable. Therefore, evaluating the limit of elements with finite size leads to the a set of countable points and not to the continuum set of points. Consequently, every point of it can not be accumulation point. This is why the following theorem holds.

Theorem 2: If we want to describe the microstructure of the micropolar material, that is the individual behavior of the microelements, then we can use only discrete models and can not constitute continuum by limitation.

The consequence of the two theorems, on the one hand, is that only one kind of contiuum—the clasical one—exist, and, on the other hand, the discrete-continuous property must be made a choice. This can be formulated as a

Theorem 3 (Theorem of choice): In the analysis of a mechanical system we must postulate whether the system is discrete or continuos.

The consequence of the proved theorems is that the continuum is only the classical one, and this exists only in the Riemann-space. The micropolar “continuum” does not exist and in the process of its contstitution the limitation was made in an illegal way.

The modelling of a physically discrete system is possible only by discrete methods.

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Mechanics of Solids,
Brainstorming Seminars at the
Budapest University of Technology and Economics

4th April 2000

LOAD-BEARING GLASS STRUCTURES

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Keywords: load-bearing glass structures.

1991 Mathematics Subject Classification: 74B99.

Summary: The lecture focused on the static design of the different load-bearing glass-structures. This topic included the physical properties of glass especially the design value of strength. Different characteristics of various types of glass, and various forms of application were also discussed.

First, loads on glass structures, for which structures are generally designed, were introduced, then special effects typical of glass were pointed out. The effect of superimposed loads was also covered. The effect of air pressure (lower or higher that that of the external air) between glass panes with thermal insulation, the reasons for that, and its numerical modelling were also presented.

The problem of safety is of great significance because of the various types of failure. A considerable difference compared to other structural materials is that in some cases equilibrium may be ensured only after cracking.

There are three possibilities for supporting a sheet glass. In case of mechanic joints, there is a great concentration of stresses, which may be avoided if a glued joint is applied. A disadvantage of the latter is the badly predictable behaviour of the glue under long-term loads. A third option is load transfer by friction, if sufficient surface area is provided. This is a reliable type of joint also for long-term loads, but it is more complex than others.

In the last part of the presentation, some examples for the application of glass structures were given, and limits of applicability were also mentioned.

Acknowledgement: I am very grateful to my advisor Gy. Deák and to K. Rákóczy for the discussions and suggestions.

Mechanics of Solids,
Brainstorming Seminars at the
Budapest University of Technology and Economics

18th April 2000

LATERAL-TORSIONAL BUCKLING OF BEAMS TAKING THE SHEAR DEFORMATION INTO ACCOUNT

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Keywords: lateral-torsional buckling, shear deformation.

1991 Mathematics Subject Classification: 74G60.

Summary: When symmetrical beams are loaded in the plane of symmetry they may deform in the symmetry plane. However, at a certain level of the applied load, the beam may buckle laterally, while the cross-sections of the beam rotate about the beam's axis. This phenomenon is called lateral-torsional buckling (or lateral buckling), and the value of the load, at which the buckling occurs, is called the buckling load or the critical load.

This problem is important if the beam's transverse bending stiffness is significantly smaller than its vertical bending stiffness. The problem of lateral-torsional buckling is even more important for lifted beams where the lifting frame does not prevent the rotation of the ends of the beam about the beam's axis.

For the lateral-torsional buckling of beams is extensively treated in several books. To the author's knowledge, none of the literatures is taking the transverse shear deformation of the beam into account. The explanation for this is that for isotropic solid or thin-walled beams the lateral-torsional buckling is important if the beam is slender, however the shear deformation in slender isotropic beams can be neglected.

Nevertheless, there are three important practical problems, when the transverse shear deformation plays a significant role in the lateral-torsional buckling.

1. Pultruted composite beams are often made of unidirectional fibers. For such beams the longitudinal Young modulus may be 20 times higher than the shear modulus, which means that the effect of shear deformation is higher by an order of magnitude than that for isotropic beams.

2. In some cases sandwich beams are loaded parallel to their face sheets. It is well known that these types of beams have large shear deformation. For sandwiches the shear deformation occur in the core.
3. Slender beams are often jointed together by frames or trusses to form a so called twin beam. Twin beams behave similarly to sandwiches. The truss plays a similar role as the core, and when the truss deforms the overall displacements of the beam will be similar to the shear displacement of a sandwich beam. Accordingly, twin beams can be modelled by sandwich beams which 'replacement shear stiffness' can be calculated from the geometry and the stiffnesses of the truss. When the truss is slender the shear deformation may supersede the bending deformation.

In the above three cases by neglecting the shear deformation we may significantly overestimate the buckling load which leads to a very unconservative design. Our aim is to determine a closed form analytical solution to calculate the critical load of symmetricaly loaded beams taking the shear deformation into account.

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Mechanics of Solids,
Brainstorming Seminars at the
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5th October 2000

ORIGAMI IN STRUCTURAL MECHANICS

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Keywords: origami, structural stability, plastic buckling, travelling plastic hinge, buckling pattern.

1991 Mathematics Subject Classification: 74G60.

Summary: Origami, that is, paper folding has a great tradition in Japan, and it is popular all over the world. However, origami is not only a recreational activity, but a scientific tool in structural engineering. It has been known for a long time that the stiffness of plates can be increased by folding. This is also true for curved surfaces. The folded cylindrical roof of a university auditorium in Hungary and Koryo Miura's coffee can in Japan are examples of that.

In the paper we investigate an application of origami in inextensional buckling forms of columns with thin-walled closed polygonal sections. These axially compressed columns loose their stability usually with local buckling [1]. Experiments show that the buckling pattern is formed with curved lines. The actual buckling patterns, because of the finite wall thickness, are in general not inextensional, but they tend to be close to an inextensional form. The buckling patterns appear only in the post-buckling phase in the plastic region. Therefore, the theoretical investigation of buckling is possible only if plasticity is taken into account. The failure mechanism of an axially loaded simply supported steel plate is a roof-like bump, where two triangular regions are considered completely plastic. This buckling mechanism, which is not inextensional, is acceptable for square box columns in some cases, but it is not proven by experiments for six- or more-sided box columns. For the latters and even for square box columns an inextensional model with travelling plastic hinges provides a better approximation of reality. Here two problems arise:

1. What inextensional deformations are possible for prismatic surfaces; can they be curved-lined?
2. How to approximate an actual buckling pattern by a polyhedron isometric to the polygonal tube?

The response to the first question is important because an improper answer can lead to incompatibility problems [2]. The answer to the second question is important from the point of view of the rigid-plastic model. Curved lines do not fit to the rigid-plastic model, therefore, a polyhedron approximation is necessary, where all the yield lines are straight line segments. Both questions are geometrical in nature. They are investigated in this paper.

Actual calculations for the plastic buckling with travelling plastic hinge model have been executed for a square box column. The results are presented.

Acknowledgement: The financial support from FKFP 0391/1997 and OTKA T031931 is hereby gratefully acknowledged.

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Mechanics of Solids,
Brainstorming Seminars at the
Budapest University of Technology and Economics

19th October 2000

MODELLING OF THE HUMAN SPINE

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Keywords: biomechanics, spine, spondylolysis, spondylolisthesis.

1991 Mathematics Subject Classification: 92C10.

Summary: One of the most important questions of the spine-biomechanics is the modelling of the spondylolysis' and spondylolisthesis' development. In the condition of spondylolysis, a crack occurs in the pars interarticularis. Associated with spondylolysis is a tendency for the vertebra involved to slip forward on the one below, which is termed spondylolisthesis.

The etiology and predisposing factors of spondylolysis and spondylolisthesis have received considerable attention, but there biomechanics is little published.

The goal of our presentation is to show a biomechanical model for the development of spondylolysis and spondylolisthesis. The facet joint is biomechanically important for at least two reasons. First, the facet joint guides the movement of the spine. The type of motion possible at any level of the spine is determined by the orientation of the facets of the transverse and frontal planes. This orientation changes throughout the spine. In the lumbar region the facet is oriented at right angles to the transverse plane and at 45-degree angle to the frontal plane. This alignment allows flexion, extension and lateral flexion but almost no rotation. The lumbosacral joints differ from the other lumbar intervertebral joints in that the oblique orientation of the facets allows appreciable rotation.

Second, the facets have a load-bearing function. The studies of King et. al. have established that the loads on the facets are greatest, about 30 the total load, when the spine is hyperextended. The facets play an important role in resisting shear force. The experiments of Miller *et al.* [3] and Adams *et al.* [4] have demonstrated that the facet joints provide about 70–85 permanent load.

For the modeling of spondylolysis and spondylolysthesis, the compressive and shear loads of the spine have to be determined taking into account the anatomical angle of the lumbrosacral joint. On the basis of our theoretical study we have established that the angle of the lumbrosacral joint plays an important role in the

development of the spondylolysis and spondylolysthesis, because the magnitude of shear and compressive loads depend on the angle of the lumbrosacral joint.

One part of the developed shear force is resisted by intervertebral discs and ligaments, and the other part by facets. The portion of forward shear resisted by the facets is assumed to be shared equally between the two facets as compressive force.

The forces acting on the neural arch are the force in the back muscles on the studied vertebra and the shear force on the facets. The problem is three-dimensional in that there are two inferior articular facets about 4 cm apart which try to bend the neural arch away from the vertebral body. However, to simplify the problem in two dimensions, the forces acting on the neural arch can be rewritten as a bending moment, a tensile force, and a shear force.

The pars interarticularis, the narrowest part of the neural arch is a structural element. The applied axial loads (tension force and bending moment) are supported primarily by internal forces in the pars interarticularis and in ligaments (ligamentum flavum, interspinosum, supraspinosum and intertransversalis). Based on the strains in the cross-section resulting from the external loads, we can establish that the ligament in the compressive part (ligamentum flavum) does not play a role in load-bearing because ligaments only resist tensile forces. The load-bearing capacity of the two ligaments near the neutral axis (ligament interspinosum and supraspinosum) are negligible. Thus, we can establish that only the pars interarticularis and the ligament intertransversalis play an important role in the load-bearing capacity of the neural arch.

With mechanical equations it has been established, the uncracked pars intertarticularis and the ligaments resist together the tensile and shear force, the bending moment. If the tensile stress in the pars interarticularis reaches its strength, crack occurs, and the spondylolysis is developed. The cracked pars interarticularis is no longer capable of sustaining tension, the tensile force is transferred to ligaments.

When the compressive strain of the pars interarticularis reaches its strain limit, the spondylolisthesis does not develop, because the vertebra can not slip with the unbroken ligaments. If the loading on the pars interarticularis would be decreasing, the pars interarticularis can ossify. If the tensile stress in the ligament reaches its strength and the ligament breaks, the vertebra slips and the spondylolisthesis develops.

The development of spondylolysis and spondylolisthesis depends greatly on the strength of the pars interarticularis and the ligaments. The other important question is the overloading of spine, which can come from hyperextensed or hyperflexed movements.

Acknowledgement: The financial support from FKFP 0366/97 and from Bolyai Stipendium is hereby gratefully acknowledged.

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Mechanics of Solids,
Brainstorming Seminars at the
Budapest University of Technology and Economics

2nd November 2000

EFFECT OF GRAIN SIZE ON THE LOAD-BEARING CAPACITY OF CONCRETE AND REINFORCED CONCRETE STRUCTURES

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Keywords: reinforced concrete, grain size, size effect, laboratory tests.

1991 Mathematics Subject Classification: 74-05, 74R10.

Summary: It is a generally accepted consideration in practice, that there is no difference between the behaviour of concrete and reinforced concrete structures even if the applied grain size distribution is different. (Grain size distribution will be characterised here by the maximum grain size, $d_{\max}$.) The same load-bearing capacity (maximum of the load) and failure mode is expected regardless of the maximum grain size if the strength of the concrete (strength class defined by codes) is the same.

According to experiments [2] there is a change in the load-bearing capacity of the concrete structure according to the change of the maximum aggregate size. Unfortunately, this is not unique. For some structure sizes it is a larger dmax that provides larger load-bearing capacity, for others, it is a smaller one. This may be a result of some scatter of the experimental data due to measurement error or strength variation of the material.

An experimental program was designed to investigate this problem. Three-point bending specimens of different aggregate sizes but of the same strength class were used. The cross-sectional size of the specimen was 10 × 14 cm and the span was 100 cm. The maximum sizes of the rounded aggregate ware 8, 16, and 32 mm. Both concrete and reinforced concrete specimens were cast, latter ones were made with 0.36 and 1.08 % reinforcement ratio. The compressive strength of the concrete was measured on concrete cubes, which showed that the concrete of each specimen was in the same strength class.

The concrete specimens had the same bending failure mode but different load-bearing capacity; the largest value was obtained for $d_{\max} = 16$ mm.

The reinforced concrete specimens had different failure modes and different load-bearing capacity. For smaller reinforcement ratios shear failure was typical. It was combined with bending failure for $d_{\max} = 16$ mm, and also with debonding cracks along the reinforcement for $d_{\max} = 32$ mm. For larger reinforcement ratios, both bending and shear cracks were developed. The latter one corresponded to the truss model for $d_{\max} = 8$ mm and to the arch model for $d_{\max} = 16$ mm, while for $d_{\max} =$ 32 mm, it was more like a secondary diagonal crack that appeared at failure. The load-bearing capacities are different and there is no unique variation with $d_{\max}$. It is also worth mentioning that the type of failure ranged from brittle to ductile, depending on the combination of the amount of reinforcement and the applied grain size.

The reason for ductile to brittle transition can be the increased reinforcement ratio and/or specimen size. However, there is no direct explanation for the non unique variation of load-bearing capacity.

We have thorough experimental and theoretical knowledge of the size effect of concrete [1]. According to that, the nominal strength of the concrete and reinforced concrete structures is reduced by increasing structure size. The reason for this is not statistical but rather deterministic: namely, the energy release due to cracking of concrete.

The size effect model is an approximate nonlinear fracture mechanics model [1]. It shows grain size dependence. Considering this fact, the non-unique variation in the load-bearing capacity can be explained, by providing a deterministic reason for the effect that is otherwise thought to be a statistical one. This explanation is valid regardless whether the grain size is scaled with the structure size or not.

Conclusion: the effect of grain size on the load-bearing capacity has a deterministic reason, and the effect depends not only on the grain size but also on the size of the structure.

Acknowledgement: The financial support from National Science Foundation (OTKA) under grant number F014931 is hereby gratefully acknowledged.

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Mechanics of Solids,
Brainstorming Seminars at the
Budapest University of Technology and Economics

16 November 2000

GRANULAR MATERIALS FROM THE VIEWPOINT OF A PHYSICIST

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Keywords: granular materials.

1991 Mathematics Subject Classification: 74E20.

Summary: Materials in a granular state are of great technological importance which has made them a subject of intense engineering research for a long time. In recent years these materials have attracted a considerable interest also in the physicist community. The aim of this talk was to give an overall picture of this class of materials as physicists see them, provide an insight into their rich phenomenology and unusual behaviour, reveal those properties which hinder the classical approaches, and present some new concepts, that physicists try to apply to describe them.

When walking on the beach, we feel sand as a solid body under our feet, but we can also pour it like a liquid, and when it is carried off by wind it behaves like a gas. The behaviour of this state of matter is extremely diverse, and though a great number of classical models are apt to describe some aspects of them, a global description is still missing.

The most fundamental property of granular matter which makes classical statistical physics fail, is that the grains are macroscopic and so thermal energy is negligable compared to the gravitational energy of the particles. Consequently the system is very far from being ergodic, it is frozen in a metastable state. This leads to hysteretic behaviour and history effects. Classical thermodynamical driving forces are dominated by microscopic mechanics leading to segregation and stratification. Classical elasticity theory fails due to the discreet nature of the grains, and also the Navier Stokes equations of hydrodynamics lose their general applicability because of the existence of specific dissipation mechanisms and the lack of a microscopic velocity scale. The observed unusual phenomena range from arching and force chains to density waves, stick-slip internal friction, vibration patterns and oscillons.

The extraordinary behavior opens free scope for novel concepts of modern physics. A well-known example is Self-Organized Criticality, which was first described in a sandpile model, but chaos theorem is also applicable in some cases. One of the most crucial steps in modern granular physics was the appearance of powerful computer simulations. And finally, our understanding of granular materials may also be enhanced by useful analogies: traffic models and thermal ratchets are presented as examples.

Our contribution to this field is best described by some of our publications.

Acknowledgement: The help and support of my advisor Prof. Tamas Vicsek is gratefully acknowledged.

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Mechanics of Solids,
Brainstorming Seminars at the
Budapest University of Technology and Economics

28th November 2000

SNOWBOARD DESIGN

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Keywords: snowboard, design, composite, large displacement.

1991 Mathematics Subject Classification: 74E30.

Summary: Interest in the sport of snowboarding has exploded since its introduction in the 1970's. While the sport's image has successfully attracted a new generation of riders to the mountains and encouraged dedicated skiers to convert, manufacturers struggle to develop innovations because they rely on a costly and time-consuming method of prototyping. An unconventional, yet favorable snowboard design might go untested within the constraints of the present method. However, this process could be greatly improved with the introduction of a software-based design tool that allows manufacturers to predict the mechanical characteristics of the snowboard and evaluate the board's performance without construction of a physical model. Inputs to the computer model would include material properties, geometry, and construction, rider characteristics and ability, as well as a snow model and slope conditions. This would allow the user to compare different snowboard designs and determine which snowboard maximizes a particular rider's performance while ensuring the rider's safety is not compromised. In this manner, a tremendous number of snowboard designs could be easily analyzed and evaluated to optimize rider performance and satisfaction.

Acknowledgement: This project is funded through the Stanford Graduate Fellowship. Thanks to the K2 Corporation for the donation snowboard designs and engineering data for comparison.

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Mechanics of Solids,
Brainstorming Seminars at the
Budapest University of Technology and Economics

5th December 2000

COMPUTER SIMULATION OF FRAGMENTATION OF BRITTLE SOLIDS

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Keywords: fracture, fragmentation, discrete elements simulations.

1991 Mathematics Subject Classification: 74K15.

Summary: Fragmentation, *i.e.* the breaking of particulate materials into smaller pieces is a ubiquitous process that underlies many natural phenomena and industrial processes. The length scales involved in it range from the collisional evolution of asteroids through the scale of geological phenomena down to the breakup of heavy nuclei. In most of the realizations of fragmentation processes the energy is imparted to the system by impact, *i.e.* typical experimental situations are shooting a projectile into a solid block, free fall impact with a massive plate and collision of particles of the same size. The most striking observation about fragmentation is that the size distribution of fragments shows power law behavior independent on the microscopic interactions and on the relevant length scales, *e.g.* the charge distribution of small nuclei resulted from collisions of heavy ions exhibits the same power law behavior as the size distribution of asteroids. Experiments revealed that the power law behavior of fragment sizes is valid for a broad interval of the imparted energy, which was also reproduced by computer simulation of sophisticated microscopic models [1–5].

Recently, we have worked out a two-dimensional dynamical model of deformable, breakable granular solids, which enables us to perform molecular dynamics simulation of fracture and fragmentation of solids in various experimental situations [1,2]. Our model is an extension of those models which are used to study the behavior of granular materials applying randomly shaped convex polygons to describe grains. To capture the elastic behavior of solids we connect the unbreakable, undeformable polygons (grains) by elastic beams. The beams, modeling cohesive forces between grains, can be broken according to a physical breaking rule, which takes into account the stretching and bending of the connections. The breaking rule contains two parameters controlling the relative importance of the stretching and bending breaking

modes, respectively. At the broken beams along the surface of the polygons cracks are generated inside the solid and as a result of the successive beam breaking the solid falls apart. The fragments are defined as sets of polygons connected by remaining intact beams. The time evolution of the fragmenting solid is obtained by solving the equations of motion of the individual polygons until the entire system relaxes, *i.e.* there is no breaking of the beams during some hundreds of consecutive time steps and there is no energy stored in deformation. For more details of the model's definition see Refs. [1,2].

We have applied the model to study shock fragmentation of solids in various experimental situations. Namely, simulations were performed to study the fragmentation of a solid disc caused by an explosion in the middle [1], the breaking of a rectangular block due to the impact with a projectile [1], and the collision of two macroscopic bodies (discs) [2]. The model proved to be successful in reproducing the experimentally observed subtleties of fragmenting systems, *e.g.* the power law mass distribution of fragments was found independent on the initial conditions, with an exponent in the vicinity of two, slightly depending on the initial energy [1,2].

Recently, simulating collisions of two solid discs we showed that, depending on the initial energy, the outcome of a collision process can be classified into two states: a damaged and a fragmented state with a sharp transition in between. We provided numerical evidence that the transition point between the two states behaves as a critical point, and our studies gave also some insight into the mechanism of the phase transition [6].

Acknowledgement: The author is grateful for the Bólyai János Fellowship of the Hungarian Academy of Sciences.

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Mechanics of Solids, 14th December 2000
Brainstorming Seminars at the
Budapest University of Technology and Economics

SOME APPLICATION OF FRACTAL GEOMETRY IN GEOTECHNICS

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Keywords: fractal, flow problem, granular soil, numerical models.

1991 Mathematics Subject Classification: 37F99.

Summary: Fractal analysis, as a result of recent developments in mathematics, now used in Geotechnics. The fractal models for granular soils are valid up to hundred times of the significant particle size, which is the scale of laboratory tests. In larger scale projects the methods of geostatistic should be applied, among them the fractal geometrical analysis is also available. One possibility is quantifying soil microstructure. The other ideas of fractals and its theoretical application for investigation on flow problems are also introduced and the methods of its numerical modelling are also discussed.

According to the Father of fractal geometry, Mandelbrot, *'Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line'*. Neither the soil is homogeneous, elastic and isotropic halfspace, as it is supposed in theoretical investigations. There is no good mathematical formula for the description of soils taking into account the shape of the particles, the phase composition, etc. The fractal analysis may give better mathematical description of soil.

There are two ways of the formation of soil. One is the disintegration of rocks; the other is the sedimentation and cohesion of grains carried by wind or water. There are mathematical fractal analogies for both processes of formation of soil (making fractals with growth pattern or erosion technique), so it is plausible to apply the fractal geometric methods in Geotechnics. In mathematics there are deterministic or random fractals. In Nature there are examples for both deterministic and random fractals. There is limited validity of the theoretical equation to the fractals in Nature; for example the range for validity for a tree is from 1 cm to 10 m. A test configuration measuring the porosity of soil is shown. The soil sample is injected slowly with liquid

Wood's metal (melting point 70°C) and cut into segments after getting cool. Image processing based on deviation of colours makes the determination of fractal dimension.

To model flows in porous medium, one possibility is the diffusion model (diffusion limited aggregation, DLA). For the modelling, making appropriate lattices and an initial layer at the injected point is required. There are meandering particles in the model sticking to the initial layer or to the thickening layer because of diffusion. I showed a perpendicular sticking model and a 45° sticking model. The diffusion model is valid in laboratory condition to highly viscous liquid. The other modelling possibility is the Eden model. Then the meandering particles move outward and stick at the boundary. This model represents a very slow filling. The third modelling possibility is the ballistic model. Then the meandering particles fall vertically and stick at the boundary. I showed the vertical sticking, the perpendicular sticking and the 45° sticking ballistic model. Of course, in the laboratory modelling other formation or the combinations of the presented mathematical models are also possible.

One very current test apparatus is the Hele-Shaw cell. The results of ballistic fractal modelling fit well to the test result.

The filling speed is the highest in DLA, but the degree of saturation is the smallest. These models maybe well for modelling pollution wash out from soil. The ballistic models slightly differ in degree of saturation and in filling velocity from DLA models. The same are true for ballistic and Eden models. The Eden model is valid when the soil has higher porosity and ballistic is valid for lower ones, when the filling velocity is the same. The ballistic models are different in available degree of saturation. These models maybe fit to the soils that have capillarity, when the filling test is made with low pressure upward.

Acknowledgement: This paper introduces the research done at the Budapest University of Technology and Economics, and at the Ybl Miklós Polytechnic partially financed by the Hungarian National Research Fund No. 022138 entitled 'New mathematical models in Geotechnics' and No. T020446 entitled 'Investigation on flow problems in unsaturated soil'.

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