MECHANICS OF SOLIDS

Brainstorming seminars at the
Technical University of Budapest

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PREFACE

There are several departments at the Technical University of Budapest dealing with theoretical or applied mechanics. The fact that sometimes there is no current of information existing between them, and people often do similar work or carry out research without being aware of the situation, seems too luxurious. Time, energy, talent and money are equally wasted.

This observation inspired two PhD students, Krisztina Polgár and Árpád Meggyes, to initiate the Solid Mechanics Seminars in 1998. This lecture series aims at giving an opportunity for PhD students and young scientists to present the results of their research to a friendly but critical audience, and to foster new ideas through the lectures given by more experienced researchers and professors. The seminars are held at the Department of Applied Mechanics and at the Department of Structural Mechanics of the Technical University of Budapest in an alternating manner to build a closer connection between our departments. The main intention was, however, and still is to draw together people working in a wide variety of interesting fields related to solid mechanics. This allowed us to learn from each other and also to initiate open discussions on the topics covered by the lectures. Our hope is that it was fruitful for all those who spent their valuable time with us either by giving presentation or by attending them. We also hope that we could connect people working on different research areas so that they could further exchange ideas and promote their scientific results. If only part of these goals have been achieved, we are satisfied, our efforts were not futile.

We are indebted to the professors of our departments for their encouragement. We are the most grateful, however, for those who volunteered to give fascinating talks of high scientific level. We also have to thank everybody who showed interest, came, and spent his time with us. We hope it was worthwhile, and we also hope that the remarks, sometimes provocative discussions, but the always warm atmosphere was also beneficial for the lecturers. We are grateful to the Research Relations Department of the Technical University of Budapest for the financial support. The publication of this proceedings of eleven abstracts would not be possible without their funding.

We hope to further increase the interest in our seminars in the future. Fortunately, we are not lacking volunteering lecturers from various topics of solid mechanics, which can attract a broad audience.

Budapest, April 2000.

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Mechanics of Solids,
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4th March 1999

FERROGEL: A MAGNETIZABLE RUBBERELASTIC MATERIAL

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Keywords: ferrogel, large deformation, magnetic body force.

1991 Mathematics Subject Classification: 74B20, 74F15.

Summary: A ferrogel is a chemically cross-linked polymer network swollen by a ferrofluid. A ferrofluid or a magnetic fluid is a colloidal dispersion of monodomain magnetic particles. The particles carry a considerable magnetic moment and are superparamagnetic at ambient temperature. In the ferrogel the finely distributed magnetic particles are located in the swelling liquid (practically water) and attached to the flexible network chains by adhesive forces.

Description of the motion or deformation of ferrogels is not an easy task and requires coupling the equations of magnetism and non-linear elasticity. Since the body force deforming the gel varies from point to point in space the produced deformation is nonhomogeneous in all cases. In our lecture we presented a mechanical model assuming that ferrogels show a simple Langevin-type magnetisation and are incompressible and the small particles do not influence the elastic characteristics of the gel that is to say a neo-Hookean strain-energy function can fully describe the elastic properties. The mechanical model developed for magnetic continuum can successfully describe the deformation in static magnetic fields. With the aid of the model, it is possible to reveal the unique characteristics of the magnetic field induced deformations (i.e. non-continuity, non-homogeneity) or predict the deformation of a highly elastic magnetic substance in a certain magnetic field. We also presented some finite element calculations carried out with the MARC program. Since magnetic forces are not built into the program we added an user subroutine to MARC, which calculates the magnetic body-force. The result of the finite element calculations is in good agreement with experimental observations and other one-dimensional simulations.

Acknowledgement: Financial support from OTKA grant (No. F 030378) is gratefully acknowledged.

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18th March 1999

ANALYSIS OF TENT STRUCTURES

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Keywords: prestressed membrane structure, cutting pattern, dinamic relaxation.

1991 Mathematics Subject Classification: 74K15.

Summary: Membrane structures prestressed by masts and rigid or cable edges are more and more often used for the covering of cultural and sport centers, exhibition halls, etc. A new method is demonstrated in this paper for the analysis of prestressed tent structures.

The method and the developed computer program includes 4 main steps. The first step is the determination of the equilibrium shape, that corresponds to the edges and the supposed stress distribution in the structure (theoretical shape). The determination of the theoretical shape is a pure geometrical problem, the material constants of the tent are not taken into account. By changing the supposed stress distribution in the structure it is possible to change the equilibrium shape. If the stress is supposed to be constant, than the area of the surface is minimal. In economic point of view this is the best shape, but sometimes the minimal shape is not one continous surface, so it is not usable as the theoretical shape of a tent structure. In this case an other stress distribution (and equilibrium shape) has to be used, for example the constant principal projected stresses always imply a continous equilibrium shape. The theoretical shape can be used for the architectural plan.

The second step of the method is the cutting pattern generation. Tent structures are usually prepared from orthotropic material stripes by welding. The cutting pattern gives the data of the stripes of cloth, that show together the surface of the tent. The cutting pattern is determinated on the basis of the theoretical shape.

The third step of the analysis is the determination of the real shape, that corresponds to the cutting pattern, material constants and edges. Because of the approximation used during the cutting pattern generation and the orthotropic behaviour of the cloth, the stress distribution and the real shape depart from the supposed stress distribution and the theoretical shape.

The final step is the structural analysis, the determination of the stresses and displacements on the basis of the real shape. In the third and fourth step the cutting

pattern is known and the real warp and weft directions of the material are taken into account. The equilibrium positions of the tent are determinated with the help of the dinamic relaxation method. During the analysis the double curved surface of the tent is approximated with plane triangular element, the material of the tent is supposed to be orthotropic.

Acknowledgement: The financial support from FKFP/0308 is hereby gratefully acknowledged.

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15th April 1999

ANALYSIS OF SPHERICAL MECHANISMS

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Keywords: foldable structure, spherical mechanism, infinitesimal motion.

1991 Mathematics Subject Classification: 37J20.

Summary: Some structural problems and numerical analysis of deployable roofs are presented. Overconstrained bar-and-joint mechanisms are well known in many areas of application. A typical example is a structure made of straight bars for covering large spaces, with a certain order of rotational symmetry [1]. In covering problems, however, a surface of double curvature gives better solution, so an attempt was made for the development of the spherical analogon of the previous (planar) structure. With the condition that the structure does not contain any sliding elements, the spherical adaptation can only be made by some new considerations:

- a planar mechanism usually does not form a mechanism on the sphere with the same topology that requires other topological solutions [2,3],
- in the case of spherical motions it seems to be useful to introduce some modifications on the computational model for kinematic analysis.

The main objective of this analysis was to determine the character of movability of the given structure (degrees of freedom, finite or infinitesimal motions). The applied numerical method was based on the analysis of the Jacobian matrix of the whole structure [4]. The following relevant problems are discussed:

- consideration of internal (scissor-like) hinges of a rigid bar,
- simplification of the 3D computational model,
- number and direction of independent motions,
- differentiation between finite and infinitesimal motions (an iterative method).

Present analysis makes possible to describe all the motions of the mechanism that shows also its practical applicability as a foldable structure.

Acknowledgement: The financial support from National Science Foundation (OTKA) under grant number T 024037, from Ministry of Culture and Education under grant number 0391/1997 is hereby gratefully acknowledged.

References: [1] Z. You and S. Pellegrino: Foldable bar structures. *International Journal of Solids & Structures* **34** (1997) 1825–1847; [2] F. Kovaćs and T. Tarnai: Foldable bar structures on a sphere. In: G. L. Balázs (Ed.): *Proceedings of 2nd International PhD Symposium*, pp. 305–311, Budapest, 1998; [3] F. Kovács: Foldable bar structures on a sphere. In: *Proceedings of IUTAM-IASS Symposium on Deployable Structures*, Cambridge, 1998 (to be published); [4] F. Kovács, I. Hegedűs and T. Tarnai: Movable pairs of regular polyhedra. In: J. C. Chilton, B. S. Choo, W. J. Lewis and O. Popovic (Eds.): *Proceedings of International Colloquium on Structural Morphology*, pp. 123–129, Nottingham, 1997.

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29th April 1999

A NEW APPROACH IN INELASTIC ANALYSIS OF STRUCTURES

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Keywords: inelasticity, simplified limit analysis.

1991 Mathematics Subject Classification: 74C10.

Summary: The mathematical theory of plasticity has been used for many years. Great contributors, such as that of R. Hill [1], J. Mandel [2], J. J. Moreau [3] and many others have helped to spread it by giving correct formulations and useful tools to solve problems.

Even if the real modelling of the behaviour of a particular material still can be discussed, the correct mathematical description of the evolution of a structure is possible for any case of loading, small or large deformations, quasi-statical or dynamical loadings.

Such a formulation is usually given in terms of rates or increments with unilateral constraints and involves very long and expensive computer calculations. Engineers have to use it commonly in the design of structures subjected to elevated temperatures and high pressures such that classical elastic response is no longer possible.

The inelastic analysis of structures consists of two essential steps:

1. The constitutive modelling of materials, based on continuum mechanics, leading to local equations.
2. The treatment of structures made of such materials, involving solution of global associated boundary value problems.

In continuum modelling of materials at least two approaches are possible:

1. A macromechanical phenomenological approach, i.e. global mathematical expressions and functions are guessed and coefficients are determined by curve fitting with some experimental tests.
2. A micromechanical physical approach when many known local micromechanical processes are incorporated into analysis.

A general review of this topic may be found in [4].

The objective of the modelling is to provide conditions under which hardening (i.e. increase of elastic limit), worksoftening (i.e. decrease of the elastic limit), creep (viscous effect), and many other classes of behaviours, have to be considered.

Having selected a particular local modelling, it is then necessary to consider the response of the structure for any given initial state and loading path. Generally, this study can only be made numerically, using computers to solve the complex differential equations. Several programs are presently available. Various numerical schemes were defined; they are usually based on the finite element method equiped with sophisticated time integration algorithms. The analysis is very long and expensive, involving incremental and step by step computations. This is even more obvious when the loading is cyclical or dynamical and when the asymptotical limiting behaviours are required.

The new analytical approach presented on the seminar has the advantage that it uses only purely elastic calculations. It bases on the introduction of the socalled structural transformed parameter. In the space of this parameter the plastic behaviour of the structure can be followed very clearly.

From the point of view of the material modelling the new approach is an intermediate one between the classical micromechanical and macromechanical approaches. It is based, as in the micromechanical approach, on the fact that the fundamental element is an aggregate of various elements but as in the phenomenological approach the local processes are not identified and the local behaviours are schematized to facilitate the determination of the global behaviour.

By the new approach it is possible to make simplified limit analysis, dynamic analysis of strustures and also contact analysis.

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13th May 1999

MECHANICAL ANALYSIS OF A HYPERELASTIC SYSTEM IN THE MEDICINE

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Keywords: strain energy, large deformation, finite element analysis.

1991 Mathematics Subject Classification: 92C10.

Summary: Because of the increasing of the medical demands in the therapy, always are stood new requirements for the technical sciences. Firstly these claims give new requirements to the applicable materials and formation of the construction. This article is about the analysis of a hyperelastic system. This system is an implant which tries to ease the situation of many people. The incontinence makes difficult the life of many people. At present there are two possibilities to handle this illness. At first the treatment by operation has been applying but it was dangerous. Now the implantation is used, because it is more reliable than the operation. The implant is a hydraulic system which is operated by patient. However the implant has some construction problems. If the shape of the implant is not acceptable then it can be painful to the patient. However we can influence the behavior of the close element of the implant which lies arround the urethra. In this article will analyse the closing process by finite element method and the deformability of the close element.

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6th October 1999

DEVELOPMENT OF THE CONSTITUTIVE OPERATOR

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Keywords: material tangent modulus, constitutive operator, elasticity, plasticity, damage, localization.

1991 Mathematics Subject Classification: 74B20, 74D10.

Summary: A more than a century long history and development of the material constitutive operator was presented. From the beginning of plastic buckling problems to the recent results of acoustic tensor of localization, the main streams of theories and research schools in the history of the constitutive operator have been considered.

Load history and stability analyses of structures with nonlinear reversible or irreversible materials leads to iteration process based on the structural tangent modulus containing the material constitutive operator. Recently the tangent operator is extended to strain softening cases, that is, damage and localization are in the focus of research. Consequently, an overview of the development of the constitutive operator and its recent modifications seems to be necessary. It was illustrated how the tangent modulus changed from a simple material constant to an indicator tensor of dissipative softening systems. The past and recent development of the constitutive operator can be surveyed by the state of art papers [1–4] representing different schools, approaches and aspects of the problem.

The concept of tangent modulus is resulted by the development of plasticity, namely, the plastic bifurcation problems, in resolving the classical Euler-problem. Plastic column buckling, the possible bifurcation of the structure was in the focus of interest at the end of the last century. Then, for a long period, the reduced modulus theory was accepted, when in the middle of this century a considerable progress has been investigated by Shanley. It took another decade until the continuum theory of bifurcation was laid down by the fundamental paper of Hill. Hill's concept of linear comparison solid has later been extended to non-associated flow law and to different hardening materials. The strain softening cases were included recently on the basis of the thermodynamics by introducing the concept of generalized time-independent

standard dissipative material, as the basis of the modern bifurcation theories. Associated and non-associated, hardening and softening or even damaging materials can be derived by means of this general, symmertic or nonsymmetric constitutive operator. If the damage leads to localization, the constitutive operator needs further modification, yielding the concept of the acoustic or localization tensor. In the classification of the bifurcation modes, diffuse (nonlocalized) and discontinuous (localized) bifurcation forms, moreover, different conditions of different types of instabilities (loss of singularity, loss of positive definiteness, loss of ellipticity or strong ellipticity) can be distinguished. A summary of the main streams of research of the constitutive operator has been presented.

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20th October 1999

SPACETIME WITHOUT OBSERVERS: CLASSICAL CONTINUUM PHYSICS

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Keywords: spacetime, continuum physics, observer.

1991 Mathematics Subject Classification: 74A05,74A20.

Summary: In the usual approach of rational mechanics the notion of observer is not introduced (see e.g. [1] and the reference therein). In kinematics people use transformation rules instead of observers. These transformation rules are defined in an ad-hoc way in different kinematic situations (for different observers). This approach results in a conceptually and formally difficult nonrelativistic spacetime model. This model is somehow implicit, because several physical concepts are hidden, do not appear explicitly in the mathematical description. This situation leads to paradoxes and uncertanities in the applications.

The motivation behind this approach of rational mechanics is to get an observer independent, objective continuum physics. In this lecture an other method is showed where an observer indepenedent nonrelativistic spacetime model results in a physically motivated and mathematical exact description. In this approach the observer is a mathamatically defined concept of the model and the usual observer (and coordinate) dependent equations of continuum mechanics can be derived from the observer independent, covariant equations for the different observers respectively [2].

At the end two particular problems were showed, as an example of the mentionede confusion where the usual approach leads to mistakes. Both problems are connected to the concept of material frame indifference:

1. Due to Noll the usual method to get observer independent constitutive equations for the balances (see e.g. at the mentioned book of Šilhavy) states that the constitutive equations cannot depend on the properties of the observer. It was demonstrated that for simple uniformly rotating observers the objective constitutive equations must depend on the observer properties [3].

2. The derivation of microstructured mumentum balance equations (see e.g. [4]) is based on an application of the transformation like observer structure of rational mechanics that uses the energy balance as a starting point to get the momentum balance. It can be shown that the underlying physical concepts behind the complicated structure cannot have the beleived wide applicability and generality.

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References: [1] M. Šilhavy: *The mechanics and thermodynamics of continuous media.* Springer Verlag, Berlin-etc., 1997; [2] T. Matolcsi: *Spacetime without reference frames* Akadémiai Kiadó, Budapest, 1993; [3] T. Matolcsi and T. Gruber: Spacetime without reference frames: An application to the kinetic theory, *International Journal of Theoretical Physics* **35** (1996) 1523–1539; [4] G. Capriz and E. G. Virga: On singular surfaces in the dynamics of continua with latent microstructure, *Quarterly of Applied Mathematics* **LII** (1994) 509–517.

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3rd November 1999

CONTACT AND THERMAL ANALYSIS OF REAL COMPOSITE-STEEL SURFACES IN SLIDING CONTACT

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Keywords: contact analysis, contact temperature, composite materials.

1991 Mathematics Subject Classification: 74A55, 74M15.

Summary: A three-dimensional elastic contact algorithm has been developed to analyse the normal contact problems of bodies having rough surfaces [1]. The algorithm can evaluate the real contact areas and contact pressure distributions using measured surface roughness data.

Following an approximate elastic-plastic contact solution the analysis produces more realistic elastic and plastic contact areas; in addition results of contact pressure distributions can be predicted according to a given maximum plastic limit pressure.

The technique can simulate (in an approximate way) the elastic-plastic sliding contact behaviour in the vicinity of asperities or concentrated contact areas by ignoring the effect of the tangential forces on the vertical displacement.

Assuming a certain sliding speed and a particular coefficient of friction the local temperature distribution due to the heat generation over the real contact areas can also be calculated for "slow sliding" problems.

The results show the moving real contact areas and the contact temperature fields for an electric spark machined steel surface moving over a planed bronze surface. Changes of the rigid body displacement, as well as the average and maximum pressures are also presented during sliding.

Numerical techniques have also been developed and used to evaluate the contact temperature distribution between real composite-steel surfaces in sliding contact [2]. To characterize the contact temperature problem of composite materials new definitions for composite Peclet numbers have been introduced. In the case of "slow sliding" problems a stationary numerical technique was applied, whereas for "intermediate and fast sliding" problems transient finite element (FE) solutions were preferred.

Contact temperature results were presented for real composite-steel sliding surfaces; the latter will help to provide information about the actual stress conditions, which are necessary to model the wear process of this pair of materials in future works.

A new version of the contact algorithm has been developed to consider the layer and substrate problem by specifying limit pressure condition in term of the thickness of TFL [3]. The results show the contact pressure distribution, the pressure maximum and the size of the contact area and the normal approach in terms of load. Another new feature of the contact algorithm improved the performance of the contact evaluation at the sliding phase of the contact deformation by introducing a linear contact pressure condition.

Asperity type contact was also detected by an experimental evaluation of the real contact area; the latter was performed by compressing a composite pin onto a glass surface. To study the sliding contact of a composite pin, a micro-sliding test was also produced by a pulling unit.

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17th November 1999

SHEARING TEST WITH CONTINUOUSLY INCREASING NORMAL STRESS

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Keywords: laboratory investigation, shearing test, Continuous Failure State, increasing normal stress.

1991 Mathematics Subject Classification: 86A60.

Summary: For triaxial tests to determine the peak and the residual strength of rock materials the International Society of Rock Mechanics standardized the "Continuous Failure State" triaxial test (CFS-triaxial test) [1]. According to this method confining pressure and axial stress are applied so as to cause the test specimen to be permanently in a state of failure. In this way it is possible with the aid of a single specimen to obtain at least parts of the failure envelope for both the peak and the residual strength. Kovári *et al.* [2] explained this method and realized that to keep the material in a pre-failure state the best is to chose the rate of the increasing confining pressure so that the slope of the axial stress-axial displacement curve is equal to the initial Young's modulus of the sample. Because the Young's modulus of a damaged material is increasing with the axial stress, one can avoid an early failure and the measured strength will have some reserves. Tisa and Kovári [3] showed that CFS-triaxial test might be directly adapted to the direct shear test, because on comparing the results of conventional triaxial tests with those of direct shear tests on joints or planes of weakness a considerable similarity may be observed. The curves representing the relationship between axial stress and axial strain in the triaxial test exhibit basically the same form as those for shear deformation and shear force in the direct shear test, including the characteristics for peak and residual strength. After several investigations they realized that using the CFS direct shear test the determination of the residual shear strength is exact but for determining the peak strength envelope of rough surfaces only with decreasing normal load is correct. According to the results of Tisa and Kovári [3] using continuously increasing normal load as CFS direct shear test for rough surfaces or those with teeth on cement mortar and brick specimens the slope of shear stress-normal stress curves was always under the curve of the exact

failure envelope which was measured with single specimens under constant normal load. Shearing tests with regular triangular teeth were carried out with increasing normal load from the maximal shearing stress point according to the research of Tisa and Kovári [3] with nine different starting normal loads and the relationship between the shear and the normal stress was described. The measured shear stress-normal stress curves were analyzed where the normal stress was increased continuously from the maximal shear stress. In this case the specimen is in "Continuous Sliding State" which is not equal to the "Continuous Failure State", as it was supposed before. The slope of the measured shear stress-normal stress curves are always independent on the starting constant normal stress. The slope of this line should depend on the roughness and the mechanical behavior of the rock and the ratio of the shear and normal stress rate. This slope is close to the teeth angle therefore with this method one measurement could be enough to determine the inclination angle. Both linear and non-linear equation was written. For the linear formula the equation of Patton [4] with theory of Ladanyi and Archambault [5] and the suggestion of Vásárhelyi [6] was used. The non-linear model is a curve fitting with the Jaeger [7] model. The slope of this line depends on the roughness and the mechanical behavior of the rock and the ratio of the shear and normal stress rate. Remarkable that the shape these curves is similar to the Ostwald curves for shear stress and shear rate [8]. The understanding of Ostwald curves are based on general thermodynamic-rheological arguments. This more general frame can give a better hope to explain the experiments than the theories supposing the failure of rigid teeth or linear elastic deformation.

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1st December 1999

DETERMINING THE SHAPE OF TWISTED RINGS

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Summary: We take an inextensible, naturally straight, long, elastic rod of circular cross-section with radius r. We bend it until it forms a ring, then twist the end sections in opposite sense. After certain a value of twist the rod lose its stability and takes a peculiar spatial form [1]. This is considered as an idealized rod model for DNA. If we don't allow the rod to come through itself, we arrive at a *contact problem*. The contact is possible either at isolated points or along intervals [2,3]. The contact force, accordingly, can be a concentrated force, or a distributed load. We show an indirect method where not the emergent configuration will be determined for a prescribed twist, but a geometrically and physically possible shape—described by one parameter—will be determined, and the twist is computed posteriorly.

We are looking for the following shape. The middle part of the configuration is a *double-helix*, where the strands have contact along a line. At both ends of the double-helix there is a contact point. Then the strands form a *terminal loop* on both ends of the contact interval.

Our supposed form has three symmetry axes, implying that the contact line is a straight line, hereafter called axis z, which is one of the symmetry axes. The rod axis forms the double-helix on the cylinder with radius r around axis z. The loop crosses axis z normal to that.

First we examine the double-helix. Its length will be denotated by parameter p. Two strands are contacting, and are symmetric to axis z. If the distributed contact force is taken into account, it is sufficient to examine one strand. This strand make a helix around axis z. The tangent of this helix close a variable angle with the plane normal to axis z. Our first goal is to determine this angle as a function of the arclength. Because of the symmetry the angle-function is an even function with respect to the middle point of the helix.

The geometrical, physical and equilibrium equations of the rod lead to a nonlinear ODE of third order. We can specify three initial values, the angle and its first and second derivatives at the middle point of the helix, where the first derivative must be zero, the others have a variable value. The IVP defined by the ODE and by these initial values will be solved numerically. At the end of the helix we compute the stresses.

After this, the loop is examined. The loop is a rod, loaded only at its ends. The rod loaded this way is called an *elastica*. The load in the starting point is the stress in the end of the helix plus the concentrated contact force. With this load, and the geometry taken from the endpoint of the helix we compute the IVP of the elastica until the half-length of loop. At this point the rod axis must cross axis z, and must be normal to that.

This condition can be represented with three equations. When the angle in the starting point, its second derivatives and the concentrated contact force have the right values (i.e. according to the parameter p), the equations are satisfied. This system of nonlinear equations has to be solved. With the solution of the two IVPs with the variables taken from the system of equations we get the twist according to a helix of length p.

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PRODUCING CURVES OF INTERACTION BY GEOMETRIC TRANSFORMATIONS

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Summary: Compressed members (e.g. columns) are often designed on the basis of curves of interaction [1]. These diagrams show the ultimate bearing capacity of the member, plotted in the space of the relevant internal forces. The diagram is constructed based on the material model (stress-strain relationship) and the shape of the cross section. The failure condition can be formulated either as the ultimate elastic stress in the extreme fibers, or as the ultimate plastic stress in all fibers, or, as the ultimate strain in the extreme fibers.

In this summary we concentrate on the first two cases and we will assume that the cross sections remain plane, also, we will assume that the cross sections have at least one plane of symmetry and the bending moment is acting in that plane.

Curves of interactions are, in general, difficult to compute. Even in the case of relatively simple shapes of cross sections and relatively simple stress-strain relationships the curve of interaction can be a highly nonlinear curve, not even being polynomial.

Instead of computing the curve of interaction for every separate cross sections and material model, we propose to use simple geometric transformations to obtain one curve from another. One of the simplest examples is when the material law is transformed in the following manner:

$$\begin{bmatrix} \sigma \\ \varepsilon \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda & 0 \\ \theta & \mu \end{bmatrix} \cdot \begin{bmatrix} \sigma \\ \varepsilon \end{bmatrix} + \begin{bmatrix} \sigma_0 \\ \varepsilon_0 \end{bmatrix} \tag{1}$$

In this case the curve of interaction will transform as

$$\begin{bmatrix} N \\ M \end{bmatrix} \Rightarrow \lambda \cdot \begin{bmatrix} N \\ M \end{bmatrix} + \begin{bmatrix} N_0 \\ 0 \end{bmatrix}, \text{ where } N_0 = \sigma_0 \cdot A \tag{2}$$

Transformation (2) is a central projection to a point on the $M = 0$ axis. Why is (1) important? Generally, two σ-ε diagrams can not be connected by (1). However, there exists an interesting special case: if we consider ideally plastic materials, then (1) connects all possible materials.

Similar transformations can be found for cases where the shape of the cross section is transformed, or, when the maximum strain provides the failure condition. The parameters of the transformation in the $[M\ N]$ plane can be expressed as functions of the parameters describing the transformations in the $[\sigma\ \varepsilon]$ or the $[x\ y]$ plane, thus this method provides direct, fast access to families of interaction curves.

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