Packing of equal circles on spherical caps

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We investigate the following problem: How must n equal circles be packed on a spherical cap of angular diameter (central visual angle) α without overlapping so that the angular radius of the circles will be as large as possible? If α is zero, the problem is reduced to finding the densest circle packing in a circle. If α is equal to 360 degrees, than the problem is identical to the Tammes problem (Fejes Tóth, 1964), that is, finding the densest circle packing on a sphere. It is apparent that if the angular diameter α varies from zero to 360º a transition from packing in a circle to packing on the sphere is obtained.

In this paper, on the basis of computer-based analysis, conjectured solutions to the problem for n = 2, 3, 4, 5, 6 will be presented for the complete range of α from zero to 360º. We will show how the packing density and the conjectured best circle configurations change with the angular diameter α of the spherical cap. The results will be given in the form of packing graphs and density diagrams.

A special emphasis will be put on the case α = 180º, that is, on the case of a hemisphere, since until now only point arrangements and not circle packings were studied on a hemisphere (Kertész, 1994). Practical importance of this problem at golf balls, geodynamic satellites, signal detecting devices, etc. will be shown.

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References

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